Some comments on "Halpern iteration for firmly type nonexpansive mappings" ¹

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Abstract

In this note we point out some major bugs appeared in [1].

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1 Introduction and Comments

In [1], Song and Chai established some results for firmly type nonexpansive mappings, however their proofs contained some major bugs.

The main result presented by Song and Chai [1] is as follows:

Theorem 1 Let $E$ be a real reflexive Banach space with a uniformly Gâteaux differentiable norm and with the fixed point property for nonexpansive self-mappings. Assume that $K$ is a nonempty closed convex

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subset of $E$ and $T : K \rightarrow K$ is a firmly type nonexpansive mapping with a fixed point. For arbitrary initial value $x_0 \in K$ and fixed anchor $u \in K$, define iteratively a sequence $\{x_n\}$ as follows:

$$x_{n+1} = \alpha_n u + (1 - \alpha_n)Tx_n.$$

Suppose that $\{\alpha_n\}$ is a sequence in $(0, 1)$ satisfying the conditions (C1) $\lim_{n \to \infty} \alpha_n = 0$ and (C2) $\sum_{n=1}^{\infty} \alpha_n = +\infty$. Then as $n \to \infty$, $\{x_n\}$ converges strongly to some fixed point $p$ of $T$.

On page 8, the following inequality holds:

(SC) $\|x_{n+1} - p\|^2 \leq \|x_n - p\|^2 - k\|x_n - Tx_n\|^2 + \alpha_n M$.

One can easily see that, with $\sum_{n=1}^{\infty} \alpha_n = +\infty$, (SC) does not imply that the sequence $\{x_n\}$ is bounded. In otherwords, if $k\|x_n - Tx_n\|^2 - \alpha_n M \leq 0$ then from (SC), we obtain that the sequence $\{x_n\}$ is not bounded.

References


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