On Generalized Continuous Maps in Čech Closure Spaces ¹

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Abstract

The purpose of the present paper is to introduce the concept of generalized continuous maps by using generalized closed sets and investigate some of their characterizations.

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1 Introduction

Generalized closed sets, briefly g-closed sets, in a topological space were introduced by N. Levine [8] in order to extend some important properties of closed sets to a larger family of sets. For instance, it was shown that compactness, normality and completeness in a uniform space are inherited by g-closed subsets. K. Balachandran, P. Sundaram and H. Maki [1] introduced the notion of generalized continuous maps, briefly g-continuous maps, by using g-closed sets and studied some of their properties.

Čech closure spaces were introduced by E.Čech in [3] and then studied by many authors, see e.g. [4], [5], [10] and [11]. In this paper, we introduce generalized closed (g-closed) sets in a Čech closure space. Generalized open (g-open) subsets of Čech closure spaces are also introduced and their properties are studied. Using the notion of g-closed sets, we introduce generalized continuous maps, which are studied too.

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2 Preliminaries

An operator $u : P(X) \to P(X)$ defined on the power set $P(X)$ of a set $X$ satisfying the axioms:

(C1) $u\emptyset = \emptyset$,

(C2) $A \subseteq uA$ for every $A \subseteq X$,

(C3) $u(A \cup B) = uA \cup uB$ for all $A, B \subseteq X$.

is called a Čech closure operator and the pair $(X, u)$ is a Čech closure space. For short, the space will be noted by $X$ as well, and called a closure space. A closure operator $u$ on a set $X$ is called idempotent if $uA = uuA$ for all $A \subseteq X$.

A subset $A$ is closed in the Čech closure space $(X, u)$ if $uA = A$ and it is open if its complement is closed. The empty set and the whole space are both open and closed.

A Čech closure space $(Y, v)$ is said to be a subspace of $(X, u)$ if $Y \subseteq X$ and $vA = uA \cap Y$ for each subset $A \subseteq Y$. If $Y$ is closed in $(X, u)$, then the subspace $(Y, v)$ of $(X, u)$ is said to be closed too.

Let $(Y, v)$ be a Čech closed subspace of $(X, u)$. If $F$ is a closed subset of $(Y, v)$, then $F$ is a closed subset of $(X, u)$.

Let $(X, u)$ and $(Y, v)$ be Čech closure spaces. A map $f : (X, u) \to (Y, v)$ is said to be continuous if $f(uA) \subseteq vf(A)$ for every subset $A \subseteq X$.

One can see that a map $f : (X, u) \to (Y, v)$ is continuous if and only if $uf^{-1}(B) \subseteq f^{-1}(vB)$ for every subset $B \subseteq Y$. Clearly, if $f : (X, u) \to (Y, v)$ is continuous, then $f^{-1}(F)$ is a closed subset of $(X, u)$ for every closed subset $F$ of $(Y, v)$.

Let $(X, u)$ and $(Y, v)$ be Čech closure spaces. A map $f : (X, u) \to (Y, v)$ is said to be closed (resp. open) if $f(F)$ is a closed (resp. open) subset of $(Y, v)$ whenever $F$ is a closed (resp. open) subset of $(X, u)$.

The product of a family $\{(X_\alpha, u_\alpha) : \alpha \in I\}$ of Čech closure spaces, denoted by $\prod_{\alpha \in I} (X_\alpha, u_\alpha)$, is the Čech closure space $\left( \prod_{\alpha \in I} X_\alpha, u \right)$ where $\prod_{\alpha \in I} X_\alpha$ denotes the cartesian product of sets $X_\alpha$, $\alpha \in I$, and $u$ is the Čech closure operator generated by the projections $\pi_\alpha : \prod_{\alpha \in I} (X_\alpha, u_\alpha) \to (X_\alpha, u_\alpha)$, $\alpha \in I$, i.e., is defined by $uA = \prod_{\alpha \in I} u_\alpha \pi_\alpha(A)$ for each $A \subseteq \prod_{\alpha \in I} X_\alpha$.

Clearly, if $\{(X_\alpha, u_\alpha) : \alpha \in I\}$ is a family of Čech closure spaces, then the projection map $\pi_\beta : \prod_{\alpha \in I} (X_\alpha, u_\alpha) \to (X_\beta, u_\beta)$ is closed and continuous for every $\beta \in I$.

**Proposition 1** Let $\{(X_\alpha, u_\alpha) : \alpha \in I\}$ be a family of Čech closure spaces and let $\beta \in I$. Then $F$ is a closed subset of $(X_\beta, u_\beta)$ if and only if $F \times \prod_{\alpha \in I} (X_\alpha, u_\alpha)$ is a closed subset of $\prod_{\alpha \in I} (X_\alpha, u_\alpha)$. 

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Proof. Let $F$ be a closed subset of $(X_\beta, u_\beta)$. Since $\pi_\beta$ is continuous, $\pi_\beta^{-1}(F)$ is a closed subset of $\prod_{\alpha \in I} (X_\alpha, u_\alpha)$. But $\pi_\beta^{-1}(F) = F \times \prod_{\alpha \in I, \alpha \neq \beta} X_\alpha$, hence $F \times \prod_{\alpha \in I, \alpha \neq \beta} X_\alpha$ is a closed subset of $\prod_{\alpha \in I} (X_\alpha, u_\alpha)$.

Conversely, let $F \times \prod_{\alpha \in I, \alpha \neq \beta} X_\alpha$ be a closed subset of $\prod_{\alpha \in I} (X_\alpha, u_\alpha)$. Since $\pi_\beta$ is closed, $\pi_\beta(F \times \prod_{\alpha \in I, \alpha \neq \beta} X_\alpha) = F$ is a closed subset of $(X_\beta, u_\beta)$.

The following statement is evident:

**Proposition 2** Let $\{(X_\alpha, u_\alpha) : \alpha \in I\}$ be a family of Čech closure spaces and let $\beta \in I$. Then $G$ is an open subset of $(X_\beta, u_\beta)$ if and only if $G \times \prod_{\alpha \in I, \alpha \neq \beta} X_\alpha$ is an open subset of $\prod_{\alpha \in I} (X_\alpha, u_\alpha)$.

### 3 Generalized Closed Sets

In this section, we introduce a new class of closed sets in Čech closure spaces and study some of their properties.

**Definition 1** Let $(X, u)$ be a Čech closure space. A subset $A \subseteq X$ is called a generalized closed set, briefly a g-closed set, if $uA \subseteq G$ whenever $G$ is an open subset of $(X, u)$ with $A \subseteq G$. A subset $A \subseteq X$ is called a generalized open set, briefly a g-open set, if its complement is g-closed.

**Remark 1** Every closed set is g-closed. The converse is not true as can be seen from the following example.

**Example 1** Let $X = \{1, 2\}$ and define a Čech closure operator $u$ on $X$ by $u\emptyset = \emptyset$ and $u\{1\} = u\{2\} = uX = X$. Then $\{1\}$ is g-closed but it is not closed.

**Proposition 3** Let $(X, u)$ be a Čech closure space. If $A$ and $B$ are g-closed subsets of $(X, u)$, then $A \cup B$ is g-closed.

**Proof.** Let $G$ be an open subset of $(X, u)$ such that $A \cup B \subseteq G$. Then $A \subseteq G$ and $B \subseteq G$. Since $A$ and $B$ are g-closed, $uA \subseteq G$ and $uB \subseteq G$. Consequently, $u(A \cup B) = uA \cup uB \subseteq G$. Therefore, $A \cup B$ is g-closed.

**Proposition 4** Let $(X, u)$ be a Čech closure space. If $A$ is g-closed and $F$ is closed in $(X, u)$, then $A \cap F$ is g-closed.

**Proof.** Let $G$ be an open subset of $(X, u)$ such that $A \cap F \subseteq G$. Then $A \subseteq G \cup (X - F)$ and so $uA \subseteq G \cup (X - F)$. Then $uA \cap F \subseteq G$. Since $F$ is closed, $u(A \cap F) \subseteq G$. Hence, $A \cap F$ is g-closed.
**Proposition 5** Let \((Y, v)\) be a closed subspace of \((X, u)\). If \(F\) is a g-closed subset of \((Y, v)\), then \(F\) is a g-closed subset of \((X, u)\).

**Proof.** Let \(G\) be an open subset of \((X, u)\) such that \(F \subseteq G\). Then \(F \subseteq G \cap Y\).

Since \(F\) is g-closed and \(G \cap Y\) is open in \((Y, v)\), \(uF \cap Y = vF \subseteq G\). But \(Y\) is a closed subset of \((X, u)\) and \(uF \subseteq G\). Hence, \(F\) is a g-closed subset of \((X, u)\).

**Proposition 6** Let \((X, u)\) be a Čech closure space. A set \(A \subseteq X\) is g-open if and only if \(F \subseteq X - u(X - A)\) whenever \(F\) is closed and \(F \subseteq A\).

**Proof.** Suppose that \(A\) is g-open and let \(F\) be a closed subset of \((X, u)\) such that \(F \subseteq A\). Then \(X - A \subseteq X - F\). But \(X - A\) is g-closed and \(X - F\) is open. It follows that \(u(X - A) \subseteq X - F\) and hence \(F \subseteq X - u(X - A)\).

Conversely, let \(G\) be an open subset of \((X, u)\) such that \(X - A \subseteq G\). Then \(X - G \subseteq A\). Since \(X - G\) is closed, \(X - G \subseteq X - u(X - A)\). Consequently, \(u(X - A) \subseteq G\). Hence, \(X - A\) is g-closed and so \(A\) is g-open.

**Proposition 7** Let \(\{(X_\alpha, u_\alpha) : \alpha \in I\}\) be a family of Čech closure spaces and let \(\beta \in I\). Then \(G\) is a g-open subset of \((X_\beta, u_\beta)\) if and only if \(G \times \prod_{\alpha \neq \beta} X_\alpha\) is a g-open subset of \(\prod_{\alpha \in I} (X_\alpha, u_\alpha)\).

**Proof.** Let \(F\) be a closed subset of \(\prod_{\alpha \in I} (X_\alpha, u_\alpha)\) such that \(F \subseteq \prod_{\alpha \in I} X_\alpha\). Then \(\pi_\beta(F) \subseteq G\). Since \(\pi_\beta(F)\) is closed and \(G\) is g-open in \((X_\beta, u_\beta)\), \(\pi_\beta(F) \subseteq X_\beta - u_\beta(X_\beta - G)\). Therefore, \(F \subseteq \pi_\beta^{-1}(X_\beta - u_\beta(X_\beta - G)) = \prod_{\alpha \in I} X_\alpha - \prod_{\alpha \in I} u_\alpha \pi_\alpha \left( \prod_{\alpha \in I} X_\alpha - G \times \prod_{\alpha \in I} X_\alpha \right)\).

By Proposition 6, \(G \times \prod_{\alpha \in I} X_\alpha\) is a g-open subset of \(\prod_{\alpha \in I} (X_\alpha, u_\alpha)\).

Conversely, let \(F\) be a closed subset of \((X_\beta, u_\beta)\) such that \(F \subseteq G\). Then \(F \times \prod_{\alpha \neq \beta} X_\alpha \subseteq G \times \prod_{\alpha \in I} X_\alpha\). Since \(F \times \prod_{\alpha \in I} X_\alpha\) is closed and \(G \times \prod_{\alpha \in I} X_\alpha\) is g-open in \(\prod_{\alpha \in I} (X_\alpha, u_\alpha)\), \(F \times \prod_{\alpha \neq \beta} X_\alpha \subseteq G \times \prod_{\alpha \in I} X_\alpha - \prod_{\alpha \in I} u_\alpha \pi_\alpha \left( \prod_{\alpha \in I} X_\alpha - G \times \prod_{\alpha \in I} X_\alpha \right)\) by Proposition 6. Therefore, \(\prod_{\alpha \in I} u_\alpha \pi_\alpha \left( (X_\beta - G) \times \prod_{\alpha \in I} X_\alpha \right) \subseteq \prod_{\alpha \in I} X_\alpha - F \times \prod_{\alpha \in I} X_\alpha = (X_\beta - F) \times \prod_{\alpha \in I} X_\alpha\).

Consequently, \(u_\beta(X_\beta - G) \subseteq X_\beta - F\) implies \(F \subseteq X_\beta - u_\beta(X_\beta - G)\). Hence, \(G\) is a g-open subset of \((X_\beta, u_\beta)\).

**Proposition 8** Let \(\{(X_\alpha, u_\alpha) : \alpha \in I\}\) be a family of Čech closure spaces and let \(\beta \in I\). Then \(F\) is a g-closed subset of \((X_\beta, u_\beta)\) if and only if \(F \times \prod_{\alpha \neq \beta} X_\alpha\) is a g-closed subset of \(\prod_{\alpha \in I} (X_\alpha, u_\alpha)\).
Proof. Let \( F \) be a g-closed subset of \((X_\beta, u_\beta)\). Then \( X_\beta - F \) is a g-open subset of \((X_\beta, u_\beta)\). By Proposition 7, \((X_\beta - F) \times \prod_{\alpha \in I} X_\alpha = \prod_{\alpha \in I} X_\alpha - F \times \prod_{\alpha \in I} X_\alpha \) is a g-open subset of \( \prod_{\alpha \in I} (X_\alpha, u_\alpha) \). Hence, \( F \times \prod_{\alpha \in I} X_\alpha \) is a g-closed subset of \( \prod_{\alpha \in I} (X_\alpha, u_\alpha) \).

Conversely, let \( G \) be an open subset of \((X_\beta, u_\beta)\) such that \( F \subseteq G \). Then \( F \times \prod_{\alpha \in I} X_\alpha \subseteq G \times \prod_{\alpha \in I} X_\alpha \). Since \( F \times \prod_{\alpha \in I} X_\alpha \) is g-closed and \( G \times \prod_{\alpha \in I} X_\alpha \) is open in \( \prod_{\alpha \in I} (X_\alpha, u_\alpha) \), \( \prod_{\alpha \in I} u_\alpha \pi_\alpha (F \times \prod_{\alpha \in I} X_\beta) \subseteq G \times \prod_{\alpha \in I} X_\alpha \). Consequently, \( u_\beta F \subseteq G \).

Therefore, \( F \) is a g-closed subset of \((X_\beta, u_\beta)\).

Proposition 9 Let \( \{(X_\alpha, u_\alpha) : \alpha \in I\} \) be a family of \v{C}ech closure spaces. For each \( \beta \in I \), let \( \pi_\beta : \prod_{\alpha \in I} (X_\alpha, u_\alpha) \to (X_\beta, u_\beta) \) be the projection map. Then

(i) If \( F \) is a g-closed subset of \( \prod_{\alpha \in I} (X_\alpha, u_\alpha) \), then \( \pi_\beta(F) \) is a g-closed subset of \((X_\beta, u_\beta)\).

(ii) If \( F \) is a g-closed subset of \((X_\beta, u_\beta)\), then \( \pi_\beta^{-1}(F) \) is a g-closed subset of \( \prod_{\alpha \in I} (X_\alpha, u_\alpha) \).

Proof. (i) Let \( F \) be a g-closed subset of \( \prod_{\alpha \in I} (X_\alpha, u_\alpha) \) and let \( G \) be an open subset of \((X_\beta, u_\beta)\) such that \( \pi_\beta(F) \subseteq G \). Then \( F \subseteq \pi_\beta^{-1}(G) = \prod_{\alpha \in I} X_\alpha \). Since \( F \) is g-closed and \( G \times \prod_{\alpha \in I} X_\alpha \) is open, \( \prod_{\alpha \in I} u_\alpha \pi_\alpha (F \times \prod_{\alpha \in I} X_\beta) \subseteq G \times \prod_{\alpha \in I} X_\alpha \). Consequently, \( u_\beta \pi_\beta(F) \subseteq G \).

Hence, \( \pi_\beta(F) \) is a g-closed subset of \((X_\beta, u_\beta)\).

(ii) Let \( F \) be a g-closed subset of \((X_\beta, u_\beta)\). Then \( \pi_\beta^{-1}(F) = F \times \prod_{\alpha \in I} X_\alpha \). By Proposition 8, \( F \times \prod_{\alpha \in I} X_\alpha \) is a g-closed subset of \( \prod_{\alpha \in I} (X_\alpha, u_\alpha) \). Therefore, \( \pi_\beta^{-1}(F) \) is a g-closed subset of \( \prod_{\alpha \in I} (X_\alpha, u_\alpha) \).

4 Generalized Continuous Maps

In this section, we introduce concept of generalized continuous maps by using the notion of generalized closed sets and investigate some of their characterizations.

Definition 2 Let \((X, u)\) and \((Y, v)\) be \v{C}ech closure spaces. A map \( f : (X, u) \to (Y, v) \) is said to be g-continuous if \( f^{-1}(F) \) is a g-closed subset of \((X, u)\) for every closed subset \( F \) of \((Y, v)\).
Clearly, if \( f : (X, u) \to (Y, v) \) is \( g \)-continuous, then \( f^{-1}(G) \) is a \( g \)-open subset of \((X, u)\) for every open subset \( G \) of \((Y, v)\).

**Remark 2** Every continuous map is \( g \)-continuous. The converse is not true as can be seen from the following example.

**Example 2** Let \( X = \{1, 2, 3\} = Y \) and define a Čech closure operator \( u \) on \( X \) by \( u\emptyset = \emptyset, u\{1\} = u\{2\} = u\{1, 2\} = \{1, 2\}, u\{3\} = \{3\} \) and \( u\{1, 3\} = u\{2, 3\} = uX = X \). Define a Čech closure operator \( v \) on \( Y \) by \( v\emptyset = \emptyset, v\{1\} = \{1, 2\}, v\{2\} = \{2\}, v\{3\} = \{3\}, v\{1, 2\} = \{1, 2\}, v\{2, 3\} = \{2, 3\} \) and \( v\{1, 3\} = vY = Y \). Let \( \varphi : (X, u) \to (Y, v) \) be defined by \( \varphi(1) = 1, \varphi(2) = 3, \varphi(3) = 2 \). It easy to see that \( \varphi \) is \( g \)-continuous but not continuous because \( \varphi(u\{1, 3\}) \nsubseteq v\varphi(\{1, 3\}) \).

**Lemma 1** Let \( (A, u_A) \) be a closed subspace of \((X, u)\). If \( G \) is an open subset of \((A, u_A)\), then \( G \) is an open subset of \((X, u)\).

**Proof.** Let \( G \) be an open subset of \((X, u_A)\). Then \( A - G \) is a closed subset of \((X, u_A)\), hence \( u_A(A - G) = A - G \). Therefore, \( u(A - G) \cap A = A - G \). Since \( A \) is a closed subset of \((X, u)\), \( u(A - G) = A - G \). Hence, \( A - G \) is a closed subset of \((X, u)\). Consequently, \( G \) is an open subset of \((X, u)\).

Regarding the restriction \( f | H \) of a map \( f : (X, u) \to (Y, v) \) to a subset \( H \) of \( X \), we have the following:

**Proposition 10** Let \((X, u), (Y, v)\) be Čech closure space and let \((H, u_H)\) be a closed subspace of \((X, u)\). If \( f : (X, u) \to (Y, v) \) is \( g \)-continuous, then the restriction \( f | H : (H, u_H) \to (Y, v) \) is \( g \)-continuous.

**Proof.** Let \( F \) be a closed subset of \((Y, v)\). Then the set \( M = (f | H)^{-1}(F) = f^{-1}(F) \cap H \) is a \( g \)-closed subset of \((X, u)\) by Proposition 4. Since \( (f | H)^{-1}(F) = M \), it is sufficient to show that \( M \) is a \( g \)-closed subset of \((H, u_H)\). Let \( G \) be an open subset of \((H, u_H)\) such that \( M \subseteq G \). Then \( G \) is an open subset of \((X, u)\) by Lemma 1. Since \( M \) is \( g \)-closed and \( H \) is a \( g \)-closed subset of \((X, u)\), \( u_H(M) = uM \cap H = uM \subseteq G \). Therefore, \( (f | H)^{-1}(F) \) is a \( g \)-closed subset of \((H, u_H)\). Hence, \( f | H \) is \( g \)-continuous.

In Proposition 10, the assumption of closedness of \( H \) cannot be removed as can be seen from the following example.

**Example 3** Let \( X = \{1, 2, 3\} \) and define a Čech closure operator \( u \) on \( X \) by \( u\emptyset = \emptyset, u\{2\} = \{1, 2\} \) and \( u\{1\} = u\{3\} = u\{1, 2\} = u\{2, 3\} = uX = X \). Let \( Y = \{a, b\} \) and define a Čech closure operator \( v \) on \( Y \) by \( v\emptyset = \emptyset, v\{a\} = \{a\} \) and \( v\{b\} = vY = Y \). Let \( f : (X, u) \to (Y, v) \) be defined by \( f(1) = f(3) = a \) and \( f(2) = b \). Then \( H = \{2, 3\} \) is not a closed subset of \((X, u)\) and \( f \) is \( g \)-continuous. But the restriction \( f | H \) is not \( g \)-continuous.
Proposition 11 Let \((X, u)\) and \((Y, v)\) be Čech closure spaces. Let \(A\) and \(B\) be closed subsets of \((X, u)\) such that \(X = A \cup B\). Let \(f : (A, u_A) \to (Y, v)\) and \(g : (B, u_B) \to (Y, v)\) be g-continuous maps such that \(f(x) = g(x)\) for every \(x \in A \cap B\). Let \(h : X \to Y\) be defined by \(h(x) = f(x)\) if \(x \in A\) and \(h(x) = g(x)\) if \(x \in B\). Then \(h : (X, u) \to (Y, v)\) is g-continuous.

**Proof.** Let \(F\) be a closed subset of \((Y, v)\). Clearly, \(h^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)\). Since \(f : (A, u_A) \to (Y, v)\) and \(g : (B, u_B) \to (Y, v)\) are g-continuous, \(f^{-1}(F)\) and \(g^{-1}(F)\) are g-closed subsets of \((A, u_A)\) and \((B, u_B)\), respectively. As \(A\) is a closed subset of \((X, u)\), \(f^{-1}(F)\) is a g-closed subset of \((X, u)\) by Proposition 5. Similarly, \(g^{-1}(F)\) is a g-closed subset of \((X, u)\). By Proposition 3, \(f^{-1}(F) \cup g^{-1}(F)\) is a g-closed subset of \((X, u)\). Therefore, \(h^{-1}(F)\) is a g-closed subset of \((X, u)\). Hence, \(h\) is g-continuous.

The following statement is obvious:

Proposition 12 Let \((X, u)\), \((Y, v)\) and \((Z, w)\) be Čech closure spaces. If \(f : (X, u) \to (Y, v)\) is g-continuous and \(g : (Y, v) \to (Z, w)\) is continuous, then \(g \circ f : (X, u) \to (Z, w)\) is g-continuous.

Proposition 13 Let \((X, u)\) be a Čech closure space and let \(\{ (Y_\alpha, v_\alpha) : \alpha \in I \}\) be a family of Čech closure spaces. Let \(f : X \to \prod_{\alpha \in I} Y_\alpha\) be a map. If \(f : (X, u) \to \prod_{\alpha \in I} (Y_\alpha, v_\alpha)\) is g-continuous, then \(\pi_\alpha \circ f : (X, u) \to (Y_\alpha, v_\alpha)\) is g-continuous for each \(\alpha \in I\).

**Proof.** Let \(f\) be g-continuous. Since \(\pi_\alpha\) is continuous for each \(\alpha \in I\), also \(\pi_\alpha \circ f\) is g-continuous for each \(\alpha \in I\).

Proposition 14 Let \(\{(X_\alpha, u_\alpha) : \alpha \in I\}\) and \(\{(Y_\alpha, v_\alpha) : \alpha \in I\}\) be families of Čech closure spaces. For each \(\alpha \in I\), let \(f_\alpha : X_\alpha \to Y_\alpha\) be a map and \(f : \prod_{\alpha \in I} X_\alpha \to \prod_{\alpha \in I} Y_\alpha\) be the map defined by \(f((x_\alpha)_{\alpha \in I}) = (f_\alpha(x_\alpha))_{\alpha \in I}\). If \(f : \prod_{\alpha \in I} (X_\alpha, u_\alpha) \to \prod_{\alpha \in I} (Y_\alpha, v_\alpha)\) is g-continuous, then \(f_\alpha : (X_\alpha, u_\alpha) \to (Y_\alpha, v_\alpha)\) is g-continuous for each \(\alpha \in I\).

**Proof.** Let \(\beta \in I\) and let \(F\) be a closed subset of \((Y_\beta, v_\beta)\). Then \(F \times \prod_{\alpha \neq \beta} Y_\alpha\) is a closed subset of \(\prod_{\alpha \in I} (Y_\alpha, v_\alpha)\). Since \(f\) is g-continuous, \(f^{-1}(F \times \prod_{\alpha \neq \beta} Y_\alpha) = f_\beta^{-1}(F) \times \prod_{\alpha \neq \beta} X_\alpha\) is a g-closed subset of \(\prod_{\alpha \in I} (X_\alpha, u_\alpha)\). By Proposition 8, \(f_\beta^{-1}(F)\) is a g-closed subset of \((X_\beta, u_\beta)\). Hence, \(f_\beta\) is g-continuous.

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