

On the positive correlations in Wiener space via fractional calculus ¹

Toufik Guendouzi

Abstract

In this paper we study the correlation inequality in the Wiener space using the Malliavin and the fractional calculus. Under positivity and monotonicity conditions, we give a proof of the positive correlation between two random functionals F and G which are assumed smooth enough. The main argument is the Itô-Clark representation formula for the functionals of a fractional Brownian motion.

2000 Mathematics Subject Classification: 60E15, 60F15, 60G05, 60G10, 60H07, 60J60.

Key words and phrases: Wiener space, Positive correlations, Malliavin calculus, Clark formula, FKG inequality, Fractional Brownian motion.

References

- [1] D. Bakry, D. Michel, *Sur les inégalités FKG*, Séminaire de Probabilités, XXVI, Lecture Note in Math. Springer, Berlin, 1526, 1992, 170-188.
- [2] D. Barbato, *FKG inequality for Brownian motion and stochastic differential equations*, Electron. Commun. Probab., 10, 2005, 7-16.
- [3] F. Biagini, Y. Hu, B. Øksendal, T. Zhang, *Stochastic calculus for fractional Brownian motion and applications*, Springer-Verlag, London Limited, 2008.
- [4] L. A. Caffarelli, *Monotonicity properties of optimal transportation and the FKG and related inequalities*, Commun. Math. Phys., 214, 2000, 547-563.
- [5] C. M. Fortuin, P. W. Kasteleyn, J. Ginibre, *Correlation inequalities on some partially ordered sets*, Commun. Math. Phys., 22, 1971, 89-103.

¹Received 26 February, 2010

Accepted for publication (in revised form) 26 May, 2011

- [6] E. Giné, C. Houdré, D. Nualart, *Stochastic inequalities and applications*, Birkhäuser-Verlag, Berlin, 2003.
- [7] G. Hargé, *Inequalities for the Gaussian measure and an application to Wiener space*, C. R. Math. Acad. Sci. Paris., 333, 2001, 791-794.
- [8] I. Herbst, L. Pitt, *Diffusion equation techniques in stochastic monotonicity and positive correlation*, Probab. Theory Relat. Fields., 87, 1991, 275-312.
- [9] C. Houdré, V. Perez-Abreu, *Covariance identities and inequalities for functions on Wiener space and Poisson space*, Ann. Probab., 23, 1995, 400-419.
- [10] Y. Z. Hu, *Itô-Wiener chaos expansion with exact residual and correlation, variance inequalities*, J. Theoret Probab., 10(4), 1997, 835-848.
- [11] S. J. Lin, *Stochastic analysis of fractional Brownian motions*, Stochastics and Stochastics Reports, 55(1-2), 1995, 121-140.
- [12] P. Malliavin, *Stochastic Analysis*, Springer, Berlin 1997.
- [13] E. Mayer-Wolf, A. S. Üstünel, M. Zakai, *Some covariance inequalities in Wiener space*, J. Funct Anal., 255, 2008, 2563-2578.
- [14] D. Nualart, *The Malliavin Calculus and Related Topics*, Probability and its applications, Springer-Verlag, 1995.
- [15] C. J. Preston, *A generalization of the FKG inequalities*, Commun. Math. Phys., 36, 1974, 232-241.
- [16] M. Sanz-Solé, *Malliavin calculus with applications to stochastic partial differential equations*, First edition, EPFL press, 2005.
- [17] H. Sugita, *Sobolev spaces of Wiener functionals and Malliavin's calculus*, J. Math. Kyoto Univ., 25(1), 1985, 31-48.
- [18] A. S. Üstünel, *An introduction to analysis on Wiener space*, Lecture Notes in Mathematics, Springer-Verlag, 1610, 1995.

Toufik Guendouzi

University Djillali Liabes

Department of Mathematics

P. O. Box 89 sidi Bel Abbes 22 000 Algeria

e-mail: tf.guendouzi@gmail.com