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Critical thinking - applied to the methodology of teaching mathematics

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Abstract

Critical thinking can be seen as having two components:

1. a set of skills to process and generate information and beliefs;
2. the habit, based on intellectual commitment, of using those skills to guide behavior.

It is thus to be contrasted with:

1. the mere acquisition and retention of information alone, (because it involves a particular way in which information is sought and treated);
2. the mere possession of a set of skills, (because it involves the continual use of them);
3. the mere use of those skills ("as an exercise") without acceptance of their results.

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1 A short history of critical thinking

The intellectual roots of critical thinking are as ancient as its etymology, traceable, ultimately, to the teaching practice and vision of Socrates 2500 years ago. His method of questioning is now known as the "Socratic questioning" and is the best known critical thinking teaching strategy.

Socrates practice was followed by the critical thinking of Plato, Aristotel and the Greek skeptics, all of whom emphasized that things are often very different from what they appear to be and only the trained mind is prepared to see through the way things look to us on the surface to the way they really are beneath the surface.

In the middle ages, the tradition of systematic critical thinking was embodied in the writings and teachings of thinkers such as Thomas Aquinas.

During the Renaissance, a flood of scholars in Europe began to think critically about religion, art, society, human nature, law and freedom. Among these scholars were Colet, Erasmus, More, Bacon.

Fifty years later in France, Descartes wrote the Rules for the Direction of the Mind, where he argued for the need for a special systematic disciplining of the mind to guide in thinking, so that every part of thinking should be questioned, doubted and tested.

The critical thinking of the Rennaissance and post Rennaissace scholars opened the way for the emergence of science and for the development of democracy, human rights and freedom of thought.

It was in the spirit of intellectual freedom and critical thought that people such as Robert Boyle and Isaac Newton did their work. In his Sceptical Chymist, Boyle severely criticized the chemical theory that preceded him. Newton, in turn, developed a far-reaching framework of thought which

roundly criticized the traditionally accepted world view.

Another significant contribution to critical thinking was made by the thinkers of the French Enlightenment: Bayle, Montesquieu, Voltaire and Diderot. They all began with the premise that the human mind, when disciplined by reason, is better able to figure out the nature of the social and political world.

In the 19th Century, critical thought was extended even further into the domain of human social life by Comte and Spencer.

In the 20th Century, our understanding of the power and nature of critical thinking has emerged in increasingly more explicit formulations.

To sum up, the tools and resources of the critical thinker have been vastly increased in virtue of the history of critical thought. Hundreds of thinkers have contributed to its development. Each major discipline has made some contribution to critical thought. Yet most educational purposes, it is the summing up of base-line common denominators for critical thinking that is most important. Let us consider now that summation.

The result of the collective contribution of the history of critical thought is that the basic questions of Socrates can now be much more powerfully and focally framed and used. In every domain of human thought, and within every use of reasoning within any domain, it is now possible to question:

- ends and objectives,
- the status and wording of questions,
- the sources of information and fact,
- the method and quality of information collection,

- the mode of judgement and reasoning used,
- the concepts that make that reasoning possible,
- the assumptions that underlie concepts in use,
- the implications that follow from their use, and
- the point of view of the frame of reference within which reasoning takes place.

In other words, questioning that focuses on these fundamentals of thought and reasoning are now baseline in critical thinking. It is beyond question that intellectual errors or mistakes can occur in any of these dimensions, and that students need to be fluent in talking about these structures and standards.

2 Critical thinking and the methodology of teaching mathematics

2.1 Critical thinking and problem solving

Promoting critical thinking and problem solving in mathematics education is crucial in the development of successful students. Critical thinking and problem solving go hand in hand. In order to learn mathematics through problem solving, the students must also learn how to think critically.

There are five values of teaching through problem solving:

- problem solving focuses the student's attention on ideas and sense making rather than memorization of facts;

- problem solving develops the student′s belief that they are capable of doing mathematics and that mathematics makes sense;
- it provides ongoing assessment data that can be used to make instructional decisions, help students succeed, and inform parents;
- teaching through problem solving is fun and when learning is fun, students have a better chance of remembering it later.

Some principles of problem solving:

The primary objective is to help the student to become aware of the fact that problem solving is not a special area but instead uses the same logical processes to which they are already familiar and use routinely. The problem statement itself is the primary cause of novice students difficulty in solving word problems. The solution is to ignore, when reading a problem statement, any phrases that start with words like "if". The initial action in starting a solution is identifying what is asked for. The student must be learned to verbalize. A verbal statement following the final result is of particular importance: what does the result tell me? In addition to completing the solution, the ending statement serves as a quick check of one's work. An adequate solution presentation does not have to be explained.

2.2 Critical thinking in the maths lesson

Mathematics is often held up as the model of a discipline based on rational thought, clear, concise language and attention to the assumption and decision-making techniques that are used to draw conclusions.

In 1938, Harold Fawcett introduced the idea that students could learn mathematics through experiences of critical thinking. His goals included

the following ways that students could demonstrate that they were, in fact, thinking critically, as they participated in the experiences of the classroom:

1. Selecting the significant words and phrases in any statement that is important, and asking that they be carefully defined.

2. Requiring evidence to support conclusions they are pressed to accept.

3. Analyzing that evidence and distinguishing fact from assumption.

4. Recognizing stated and unstated assumptions essential to the conclusion.

5. Evaluating these assumptions, accepting some and rejecting others.

6. Evaluating the argument, accepting or rejecting the conclusion.

7. Constantly reexamining the assumptions that are behind their beliefs and actions.

Fifty years later, the critical thinking is still present in the goals, but it has been subsumed by more holistic notions of what it means to teach, do and understand mathematics. The students will be able to:

1. Organize and consolidate their mathematical thinking through communication;

2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;

3. Analyze and evaluate the mathematical thinking and strategies of others;

4. Use the language of mathematics to express mathematical ideas precisely.

These ideas are very similar to those promoted by Fawcett in 1938. Little has changed in the mainstream ways that people tend to define critical thinking in the context of mathematics education. Students are expected to search for the strengths and weaknesses of each and every strategy offered. It is no longer good enough to reach an answer to a problem that was posed. Now, students are cajoled into communicating their own ideas well, and to demand the same communication from others. A shift has occured from listing skills to be learned toward attributes of classrooms that promote critical thinking as part of the experience of that classroom.

Such a class to promote critical thinking can be created by providing the conditions for the students to communicate with one another in order to reflect together on the solution to the problem. The first condition is for the students to feel free in expressing their ideas. Then, they must be able to listen attentively to their classmates and show interest in their ideas. So, they communicate both for learning mathematics and in mathematical terms. On the other hand, the students get accustomed to group work which implies mutual help and cooperation for a mutual aim.

2.3 The experiment achieved in class

The experiment was developed throughout a whole school year. The two groups (the one to which the experiment was applied and the one for check-up) were two graduating class having the same vocational profile, homogeneous enough as far as their level of mathematical training is concerned.

With one class I worked in the traditional way, using mostly the direct method. With the experimental class, group work was most often used. The groups were made up of 4 students. Each group was given a handout with problems to solve together. While working on it, the members of each group talked together, helped one another and appealed to the teacher when they needed help. Their endeavors and interest in understanding both the data and the requirements of the problem could be noticed, and they were all preoccupied by the understanding of the solution. The students worked together for the whole class. The ones with better knowledge of maths were helping the others. In the case of the good students, explaining the solution to their classmates engenders a deeper understanding; on the other hand, the other students seem to reach the understanding of the solution easier when the explanation comes from a pair than when it is the teacher who is giving it.

With the other group, where one student was required to solve the problem on the black-board, a lack of interest was noticed: the rest of the students were merely copying the solution, without trying to think of the problem themselves.

On the average, the same number of problems were solved during one class with each of the two groups, but those were encouraged to express their ideas freely, to think, to formulate hypotheses, to listen to one another and mutually estimate their judgements, had better results in the graduation exam. Besides, the math classes were very enjoyable and attractive for the students of this group.

3 Conclusions

It is undoubtedly easier for the teacher to teach the maths lesson the traditional way, the way has been doing it for years - with the black-board, the chalk, endless explanations and repetition of the same algorithms.

This paper is a challenge for the colleagues, an urge for them to try something else. Of course it is not easy and neither does it happen to us to have the students develop critical thinking from the first attempt. The process takes time and experience from both the teacher's and the students part. It is also required a very thorough preparation of the lesson, specially designed materials and mobile classroom furniture. The more homogeneous the groups are, the sooner they will be successful.

I personally consider these lessons particularly efficient, both from the point of view of the learning of maths, as well as considering the development of their skills in analyzing, reasoning, decision-making, communicating and, last but not least, listening to others and cooperating for the achievement of a mutual goal.

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